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Isotope shift studies in the first spectrum of dysprosium: confirmation of assignments to 4f^{10}5d6p and new assignments to 4f^{9}5d6s6p configuration

B.K. Ankush, A. Venugopalan, and S.A. Ahmad[a]

Spectroscopy Division, Bhabha Atomic Research Centre, Mumbai, 400 085, India

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Abstract. Isotope shifts $\Delta\sigma$ (^{160}Dy-^{164}Dy) are reported for 221 spectral lines of the neutral dysprosium atom (Dy I) in the region 470–565 nm using a photoelectric recording Fabry-Perot spectrometer and highly enriched isotopic samples of ^{160}Dy and ^{164}Dy excited in liquid-nitrogen–cooled hollow-cathode lamps. Isotope shift data for 173 of these lines are being reported for the first time. Using the isotope shift data, term isotope shifts ΔT (^{160}Dy-^{164}Dy) have been evaluated for 99 even- and 68 odd-parity energy levels of Dy I. New ΔT values have been obtained for 24 odd and 36 even levels. These new ΔT values have enabled us to check some of the existing tentative assignments and also to suggest configuration assignments to a few unassigned energy levels. The earlier tentative assignments of $4f^{10}5d6p$ configuration to many high odd levels lying above 35000 cm^{-1} have been presently confirmed. We could assign the $4f^{9}5d6s6p$ configuration to 24 unassigned even levels.

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1 Introduction

The first detailed analysis of the spectrum of neutral dysprosium (Dy I) was reported by Conway and Worden [1] who reported 141 even- and 197 odd-energy levels of Dy I from the analysis of 22000 spectral lines of dysprosium recorded in the 2300–11 400 Å region. 48 odd levels reported in [1] have not been confirmed in the later studies. Camus and Masmoudi [2] reported 51 even and 30 odd levels using their data on absorption spectra of Dy I. Wyart [3–5] carried out the analysis of the Dy I spectrum in the 2520–9090 Å region which resulted in the identification of 207 even- and 138 odd-parity new energy levels of Dy I; he also assigned many of the odd energy levels of Dy I to $4f^{9}5d6s^{2}$, $4f^{9}5d^{2}6s$ and $4f^{10}6s6p$ configurations and even levels to $4f^{10}6d6s$, $4f^{9}6p6s^{2}$ and $4f^{9}5d6s6p$ configurations based on parametric calculations. The NBS monograph by Martin *et al.* [6], where references to the earlier work on analysis of the Dy I spectrum can be found, lists 399 even and 322 odd energy levels of Dy I.

Earlier references (up to 1983) on studies of isotope shifts in the Dy I spectra can be found in our publications [7,8]. In recent years various laser techniques have been employed for accurate measurements of isotope shifts in the spectral lines of Dy I. Measurements of isotope shifts and hyperfine structures were carried out in the 4212 Å line of Dy I using the technique of collinear fast beam laser spectroscopy, involving stable as well as radioactive isotopes in the sequence between ^{146}Dy and ^{164}Dy including the high-spin isomer ^{147}Dy by Neugart *et al.* [9] which enabled them to evaluate the nuclear spins, moments and the isotopic variation of the nuclear mean square charge radii $\delta\langle r^2\rangle$. Childs *et al.* [10] have measured hyperfine structures in a few lines of Dy I in the 5576–6670 Å spectral region by Doppler-free laser-induced fluorescence and laser-rf double-resonance method. Employing the atomic beam laser fluorescence technique, Pfeufer *et al.* [11] have reported accurate isotope shifts involving stable isotopes of ^{A}Dy ($A = 160$, 161, 162, 163 and 164) in 12 lines of Dy I. Using the same technique, Wakasugi *et al.* [12] have carried out isotope shift measurements in 4 lines of Dy I and evaluated $\delta\langle r^2\rangle$ involving all the stable isotopes. Lipert and Lee [13] have recently reported isotope shifts in one line of Dy I at 8326.1 Å using a diode laser.

In our earlier publications [7,8] we have reported isotope shifts $\Delta\sigma$ (^{160}Dy-^{164}Dy) in 299 lines of Dy I covering the spectral region 3860–4700 Å and term isotope shifts ΔT (^{160}Dy-^{164}Dy) were reported for 119 even- and 97 odd-parity energy levels. A large number of unassigned even levels lying between 29000 cm^{-1} and 39800 cm^{-1} were assigned to the $4f^{9}5d6s6p$ configuration and a few levels to $4f^{9}6p6s^{2}$ and $4f^{10}6s6d$ configurations. Two new configurations, $4f^{9}5d^{2}6p$ and $4f^{10}6p^{2}$, were also suggested by us for some of the levels on the basis of their ΔT

[a] e-mail: saahmad@magnum.barc.ernet.in

values and observed intensities of transitions from the levels involved [7,8].

Out of 399 even and 322 odd known energy levels of Dy I, most of the levels lying above 28000 cm^{-1} do not have configuration assignments ([6] and references therein) except for the ones assigned by us in our earlier studies [7,8]. As for the known odd levels of Dy I between 34600 cm^{-1} and 40030 cm^{-1}, few levels have been tentatively assigned to the $4f^{10}5d6p$ configuration [6]. The present study of the isotope shift $\Delta\sigma$ (^{160}Dy-^{164}Dy) in the spectral lines of Dy I was taken up with three objectives; firstly to evaluate the term isotope shift ΔT (^{160}Dy-^{164}Dy) for the high-lying energy levels of Dy I and check the configuration assignments made on the basis of parametric calculations by Wyart [3,14], secondly to confirm the existing tentative configuration assignment of $4f^{10}5d6p$ to a large number of odd levels and lastly to suggest possible configurations to many unassigned high-lying even levels.

2 Experimental

Isotope shifts $\Delta\sigma$ (^{160}Dy-^{164}Dy) have been measured in 221 spectral lines of Dy I in the region 4700–5655 Å on a photoelectric recording Fabry-Perot spectrometer using highly enriched isotopes ^{160}Dy (78%) and ^{164}Dy (98%) in liquid-nitrogen–cooled hollow-cathode lamps, excited either as single isotope in separate lamps or as a mixture of two isotopes in one lamp. The measurements were carried out using etalons of 10, 12 and 20 mm thickness. Most of the lines were recorded with the single isotope in order to confirm the isotopic structure and also to measure the separation of close-by lines for which same wavelength values are given in [3]. This enabled us to identify and measure the exact wavelength separation of close-by lines and also measure the isotope shifts in these lines. As an example the structure of two close-by lines at 4812.676 Å and 4812.800 Å recorded with a mixture of ^{160}Dy and ^{164}Dy and also with the single isotope ^{164}Dy is presented in Figure 1a and 1b, respectively. The other experimental details are the same as given in [7].

3 Results and discussions

Isotope shifts $\Delta\sigma$ (^{160}Dy-^{164}Dy) for the 221 spectral lines of Dy I in the region 4700–5655 Å are presented in Table 1. Earlier isotope shift values are available for 48 of these lines [15–17] and in most of the cases these values agree with ours; in certain cases the earlier values have been marginally revised. For the line at 5274.068 Å, Ross [16] has measured $\Delta\sigma$ (^{160}Dy-^{164}Dy) = −61.0 mK, whereas our value is −68.4 mK. Energy level classifications are available for only 164 of these 221 lines. The remaining 57 unclassified lines have been listed as Dy I lines by Wyart [3]. We have therefore included these 57 unclassified lines of Dy I in Table 1; the measured isotope shift values in these lines should be helpful in the future energy level classification of these lines. The line

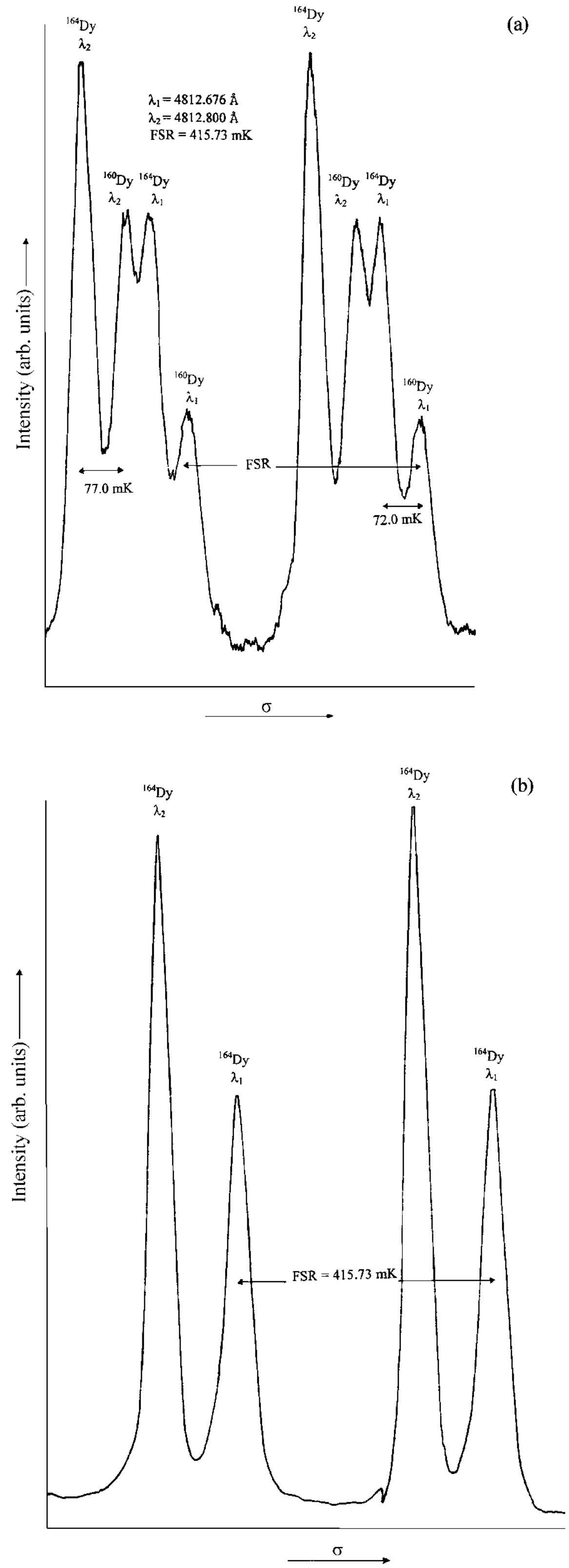

Fig. 1. (a) Isotope shift in the Dy I lines at 4812.676 Å and 4812.800 Å recorded on REFPOS using a mixture of enriched isotopes ^{160}Dy (78%) and ^{164}Dy (98%) in a liquid-nitrogen–cooled hollow-cathode operating at 32 milliamperes; (b) same two lines recorded with the single isotope ^{164}Dy under the same experimental conditions.

at 5027.874 Å has been classified by Wyart [3] as 38356 $(J = 8) \rightarrow 18472\,(J = 8)$. With this classification the expected isotope shift, $\Delta\sigma(^{160}\text{Dy}-^{164}\text{Dy})$, in this line should be about −120 mK whereas the observed isotope shift is −52.3 mK indicating that the classification needs revision. The wavelengths and the classifications, respectively in column 1 and 2, are taken from Wyart [3]. Isotope shift $\Delta\sigma$ (^{160}Dy-^{164}Dy) values presently measured by us are given in column 3 in units of milliKayser (1 mK = 10^{-3} cm^{-1}). In general the accuracy of the measurement is about 3 mK, but in favourable cases where the line intensity is good and the isotope shift is more than 50 mK the accuracy is better.

3.1 Term isotope shift ΔT (^{160}Dy-^{164}Dy) for the energy levels of Dy I

Using the isotope shift data for the classified lines of Dy I (Tab. 1), term isotope shifts ΔT (^{160}Dy-^{164}Dy) have been evaluated for 99 even- (Tab. 2) and 68 odd-parity (Tab. 2) energy levels of Dy I. The ΔT values are expressed relative to the assumed ΔT value of Z mK for the ground level of Dy I, that is ΔT ($4f^{10}6s^2$, 0.0 cm^{-1}, $^5I_8$) = Z mK, as done by us earlier [7,8]. Wyart [3] has given ΔT (^{162}Dy-^{164}Dy) values for 90 even and 69 odd levels of Dy I relative to the assumed shift ΔT ($4f^95d6s^2$, 9990 cm^{-1}, $^7I_9$) = X mK; he has also reported ΔT ($4f^{10}6s^2$, 0.0 cm^{-1}, $^5I_8$) = $(X - 37.7)$ mK. Using this relation we have converted all the ΔT values of Wyart [3] to ΔT values relative to the ground-state isotope shift. These converted ΔT values of Wyart [3] have been further converted to ΔT (^{160}Dy-^{164}Dy) using the accurate relative isotope shift values given by Dekker *et al.* [18]; these ΔT values were then normalized relative to Z mK (our assumed ΔT value for the ground state $^5I_8$ of Dy I). In the following discussions, wherever we mention ΔT values of Wyart [3], they refer to these converted ΔT (^{160}Dy-^{164}Dy) values normalized relative to the isotope shift (Z mK) of the ground state.

3.1.1 ΔT values of low even-parity energy levels of Dy I

There are 16 low even-parity energy levels encountered in the present study and these are listed in Table 2a. It was not possible to evaluate ΔT values relative to Z directly for most of the levels of the $4f^{10}5d6s$ configuration from the present isotope shift data. For the 18937 cm^{-1} level of the $4f^{10}5d6s$ configuration the ΔT value given by Wyart [3] is $(Z - 85)$ mK; we have assumed this ΔT value for the level at 17613 cm^{-1} belonging to the $4f^{10}5d6s$ configuration. For the levels at 18903 cm^{-1} and 20193 cm^{-1} Wyart [3] has given ΔT values, respectively, as $(Z - 66)$ mK and $(Z - 70)$ mK which we have used in the present work. For the level at 19240 cm^{-1} we have assumed a ΔT value of $(Z - 70)$ mK based on the ΔT value of the level at 20193 cm^{-1}, whereas for the level at 19797 cm^{-1} we have assumed the ΔT value as $(Z-70)$ mK based on the ΔT value for the level at 20209 cm^{-1}.

Table 1. Isotope shift $\Delta\sigma$ (^{160}Dy-^{164}Dy) measured presently in the lines of the Dy I spectrum in 4700–5655 Å region. The wavelengths of all these lines, listed as Dy I, and their classifications are from [3].

Wavelength (Å)	Classification Odd level	Even level	$\Delta\sigma(^{160}\text{Dy-}^{164}\text{Dy})^{\text{a}}$ (mK) (1 mK = 10^{-3} cm^{-1})
1	2		3
4703.471	15567	36822	+144.2
4703.471			+76.3
4715.493	14153	35354	∼ 0
4717.158	13495	34689	−93.3
4721.225			−70.4
4721.225			∼ 0
4724.223	17513	38674	∼ 0
4726.764	15567	36717	∼ +10
4728.869	15567	36708	+64.0
4729.087	39602	18462	∼ 0
4738.466	12655	33753	−87.6
4740.925	16733	37820	∼ 0
4741.538			∼ 0
4742.397			∼ 0
4742.698	12007	33086	−82.1
4745.590			−90.1
4745.729	18711	39777	−87.0
4761.306			∼ 0
4763.639			+62.4
4764.652	15972	36954	+52.3
4766.480	7565	28539	−72.1
4771.937	38563	17613	∼ 0
4775.789	15972	36905	∼ 0
4781.814			∼ 0
4788.392	25012	4134	∼ 0
4791.289	24999	4134	−62.2
4800.639	15567	36392	∼ 0
4802.008			∼ 0
4804.511			∼ +20
4807.942	7565	28358	−75.0
4808.738	40030	19240	∼ 0
4810.275	38297	17514	∼ 0
4812.676	24906	4134	−72.0
4812.800	8519	29291	−77.0
4814.150	20766	0	−58.8
4818.202	9990	30739	−81.5
4819.041	15972	36717	∼ +10
4821.293			∼ 0
4823.722	9990	30716	−73.0
4824.963	39182	18462	∼ −20
4828.880			−81.2
4832.376	38202	17514	∼ 0
4840.169			−77.2
4845.620			−30
4845.778	16693	37324	∼ 0
4850.849	9990	30600	−65.7
4851.435	16733	37339	∼ 0
4852.516	16693	37295	$-\varepsilon$
4855.577	38202	17613	∼ 0
4857.194	41656	21074	+64.0
4859.149			∼ 0
4865.681			$+\varepsilon$

Table 1. continued

Wavelength (Å)	Classification Odd level	Even level	$\Delta\sigma(^{160}\text{Dy-}^{164}\text{Dy})^{a}$ (mK) (1 mK = 10^{-3} cm^{-1})
1	2		3
4872.476	15972	36490	−56.0
4875.926			−87.3
4880.160	20485	0	+81.8
4884.004	16693	37163	∼ 0
4884.152	9990	30459	−74.7
4884.551			∼ 0
4886.144			∼ 0
4888.081	8519	28971	−78.9
4893.676			+10
4895.850	15972	36392	∼ 0
4897.366			∼ 0
4899.244			∼ 0
4901.940	7565	27959	−72.9
4903.682	10088	30475	−67.3
4906.251	27427	7050	−63.9
4907.478	15567	35938	+77.0
4909.798			∼ 0
4911.166	10088	30444	−74.9
4914.729	20341	0	−65.8
4916.409	18021	38356	−57.2
4917.174	7565	27896	−75.9
4918.241			−70.5
4919.553	16693	37015	$-\varepsilon$
4931.031			∼ +20
4932.473			∼ +20
4937.388	38342	18094	+52.9
4940.424	29447	9211	∼ 0
4941.158	40030	19797	−68.2
4942.849			∼ −25
4948.216	14625	34829	−83.1
4949.319	12007	32206	−70.4
4953.365	15194	35377	−79.3
4959.587			∼ 0
4959.587			∼ 0
4959.656			∼ 0
4961.741	27199	7050	−53.1
4963.825	27190	7050	∼ −10
4969.858	9990	30106	−76.7
4971.768	38202	18094	∼ 0
4975.212	12298	32392	−65.2
4980.125	16733	36807	+49.8
4981.963	16693	36760	$+\varepsilon$
4985.523			−85.3
4991.749	16733	36760	$+\varepsilon$
4992.642	17727	37751	+30
4993.517	8519	28539	−72.4
4994.808	19557	39573	−99.0
4998.461	11673	31674	−69.8
5003.867	17727	37706	∼ +10
5005.887			∼ +10
5006.267			∼ 0
5010.605	19797	39750	−122.3
5012.156			∼ 0
5012.579	10088	30033	−73.6
5019.319			∼ 0

Table 1. continued

Wavelength (Å)	Classification Odd level	Even level	$\Delta\sigma(^{160}\text{Dy-}^{164}\text{Dy})^{a}$ (mK) (1 mK = 10^{-3} cm^{-1})
1	2		3
5022.118	24040	4134	+81.1
5024.535	7565	27462	−67.3
5027.874	(18472	38356)[b]	−52.3
5032.659	17727	37591	$+\varepsilon$
5032.996			∼ +10
5039.045	8519	28358	−72.5
5042.634	7565	27390	−76.4
5045.290	16693	36508	+77.0
5045.749	12007	31820	−73.0
5047.248	8519	28326	−61.7
5050.214			∼ 0
5050.215			∼ 0
5052.009	10088	29877	−70.1
5053.188			−101.3
5053.352			−66.6
5055.462	19797	39573	−94.1
5059.957	14153	33911	−73.4
5060.732			−69.1
5061.995			$-\varepsilon$
5063.434	19304	39048	−69.7
5063.541	23877	4134	−30
5065.102	12655	32392	−72.9
5065.536	18528	38264	∼ −15
5070.677	9990	29706	−77.1
5077.669	19688	0	+84.6
5092.295			∼ 0
5097.307	17727	37339	∼ 0
5102.349	10088	29682	−71.0
5105.331	18472	38054	∼ 0
5106.882	37090	17514	∼ 0
5108.453	18021	37591	∼ 0
5110.321	19557	39120	−71.4
5111.976	38019	18462	∼ 0
5112.712	23687	4134	−51.0
5120.036			∼ 0
5128.291	7565	27059	∼ 0
5130.178			∼ −20
5132.221			∼ 0
5133.602	9990	29465	−75.4
5135.022	8519	27987	−66.0
5141.523			$-\varepsilon$
5146.978	10088	29512	−68.3
5147.192			−63.3
5155.304	17513	36905	∼ 0
5156.805	18433	37820	−88.6
5165.338	18339	37694	∼ 0
5169.583			∼ +20
5171.919	23464	4134	+62.3
5175.078	18021	37339	∼ 0
5175.187	18433	37751	−57.5
5184.505	7565	26848	∼ 0
5185.158			∼ 0
5192.495			∼ 0
5196.814	17727	36964	∼ 0
5205.663			−35

Table 1. continued

Wavelength (Å)	Classification Odd level	Classification Even level	$\Delta\sigma$(^{160}Dy-^{164}Dy)[a] (mK) (1 mK = 10^{-3} cm^{-1})
1	2		3
5205.736	17513	36717	$\sim +10$
5221.980	18711	37856	~ 0
5226.919	36640	17514	$+\varepsilon$
5236.253	19092	0	+82.9
5238.371	26135	7050	+80.7
5259.879	38247	19240	−56.7
5260.557			−71.1
5267.110	9990	28971	−81.3
5274.068	8519	27474	−68.4
5277.683	18021	36964	~ 0
5277.882	39135	20193	−65.8
5282.072	8519	27445	−53.9
5290.034	10088	28987	~ 0
5292.713			+69.7
5297.913	7565	26435	−130.0
5300.883	7565	26425	−97.3
5301.575	18857	0	+74.7
5309.886	40605	21778	+50.0
5311.849	10088	28909	−58.0
5312.627	37721	18903	~ 0
5316.980	11673	30475	−69.9
5322.232	7565	26349	−72.1
5335.053	18021	36760	$\sim +10$
5340.300			−62.7
5352.114	7565	26244	−74.8
5370.588	12007	30621	−73.8
5376.099	37836	19240	~ 0
5381.357	10088	28666	−40.0
5389.744	9990	28539	−75.4
5392.043	8519	27059	~ 0
5395.572	18528	0	~ 0
5404.191	22633	4134	+80.3
5409.680	37721	19240	$\sim +20$
5419.132			−71.9
5420.770	9990	28433	−77.7
5423.319	18433	0	$\sim +20$
5424.273	35945	17514	−48.9
5451.108	18339	0	$\sim +10$
5473.062	8519	26785	~ 0
5481.637	10088	28326	−65.0
5486.382	36316	18094	~ 0
5486.833	10088	28309	~ 0
5496.829	37090	18903	~ -20
5497.288	15567	33753	+70.0
5499.551	36640	18462	+41.6
5502.793	9990	28158	−76.3
5528.012	12655	30739	−79.4
5535.215	39135	21074	−87.8
5542.195	38247	20209	−54.3
5547.267	18021	0	−67.4
5554.691	28923	10925	~ 0
5583.194	8519	26425	−95.0
5600.800	37090	19240	$+\varepsilon$
5609.865	12655	30475	−73.6
5613.228	38019	20209	~ 0

Table 1. continued

Wavelength (Å)	Classification Odd level	Classification Even level	$\Delta\sigma$(^{160}Dy-^{164}Dy)[a] (mK) (1 mK = 10^{-3} cm^{-1})
1	2		3
5627.489	21899	4134	−67.9
5639.497	17727	0	−69.5
5645.990	35221	17514	~ -20
5652.008	17687	0	+78.5

[a] $\Delta\sigma$ less than 10 mK is indicated by ε.

[b] The observed isotope shift in this line does not agree with the classification given in [3].

These assumptions are based on the fact that the multiplet with same leading percentages and compositions would have similar ΔT values. For the level at 18462 cm^{-1} ($J = 10$) we have evaluated $\Delta T = (Z - 88)$ mK and this level has 90% purity; so for the levels at 17514 cm^{-1} and 18094 cm^{-1} we have assumed $\Delta T = (Z - 85)$ mK as these have same leading percentages. For the levels at 20209 cm^{-1} and 21074 cm^{-1} of the $4f^{10}5d6s$ configuration the evaluated ΔT values are $(Z - 73)$ mK and $(Z - 48)$ mK, respectively.

3.1.2 High even levels and their configuration assignments

We have evaluated ΔT (^{160}Dy-^{164}Dy) for 83 high-lying even levels of Dy I lying above 26000 cm^{-1} (Tab. 2b); for 36 levels ΔT values are being reported for the first time. Out of these 83 even levels above 26000 cm^{-1} (Tab. 2b), only 5 levels have definite configuration assignments and 11 levels have only tentative assignments [6]. For 19 levels (Tab. 2b) we had assigned the $4f^95d6s6p$ configuration [7, 8]; these are shown in parentheses in column 3 of Table 2b and most of these assignments have been confirmed by the parametric calculations of Wyart [14].

The average ΔT values for the known even configurations of Dy I, deduced on the basis of our present and earlier studies as well as other studies [3,7,8,15,16], can be summarized as follows:

$$\Delta T(4f^{10}6s^2) \sim Z \text{ mK},$$
$$\Delta T(4f^{10}5d6s) \sim (Z - 80) \text{ mK},$$
$$\Delta T(4f^96s^26p) \sim (Z + 110) \text{ mK},$$
$$\Delta T(4f^95d6s6p) \sim (Z + 15) \text{ mK},$$
$$\Delta T(4f^95d^26p) \sim (Z - 150) \text{ mK}.$$

$4f^95d6s6p$ configuration

The levels at 26244 cm^{-1} ($J = 9$), 26349 cm^{-1} ($J = 8$) and 26425 cm^{-1} ($J = 7$) have been tentatively assigned to the $4f^95d6s6p$ configuration by Martin *et al.* [6]. Our ΔT values for these levels conclusively show that these levels belong to the $4f^95d6s6p$ configuration and are also confirmed by Wyart's calculation [14].

Table 2. Term isotope shift, ΔT (^{160}Dy-^{164}Dy) in the even parity energy levels of Dy I encountered in the present study. Energy levels, their J values and configurations are from [3]. Z is the assumed isotope shift (ΔT) in the ground state ($4f^{10}6s^2$, 0.0 cm^{-1}, $^5I_8$); the estimated value of Z is $\sim$ 155 mK.

Energy level (cm^{-1})	J	Configuration$^+$	ΔT (mK)	Configuration (present work)
a.				
0.0	8	$4f^{10}6s^2$	$(Z)^a$	
4134.23	7	$4f^{10}6s^2$	$Z+2$	
7050.61	6	$4f^{10}6s^2$	Z	
9211.58	5	$4f^{10}6s^2$	Z	
10925.25	4	$4f^{10}6s^2$	Z	
17514.50	9	$4f^{10}5d6s$	$\sim(Z-85)^a$	
17613.36	8	$4f^{10}5d6s$	$\sim(Z-85)^a$	
18094.52	7	$4f^{10}5d6s$	$\sim(Z-85)^a$	
18462.65	10	$4f^{10}5d6s$	$Z-88$	
18903.21	8	$4f^{10}5d6s$	$(Z-66)^b$	
19240.82	9	$4f^{10}5d6s$	$(Z-70)^a$	
19797.96	10	$4f^{10}5d6s$	$(Z-70)^a$	
20193.60	8	$4f^{10}5d6s$	$(Z-70)^b$	
20209.00	9	$4f^{10}5d6s$	$Z-73$	
21074.20	7	$4f^{10}5d6s$	$Z-48$	
21778.43	7	$4f^{10}5d6s$	$(Z-86)^b$	
b.				
26244.60	9	$4f^95d6s6p$?	$Z+5$	$4f^95d6s6p$
26349.49	8	$4f^95d6s6p$?	$Z+5$	$4f^95d6s6p$
26425.15	7	$4f^95d6s6p$?	$Z-15$	$4f^95d6s6p$
26435.71	7	$4f^{10}5d6s$	$Z-51$	
26785.45	6	$4f^96s^26p$	$Z+82$	
26848.46	7	$4f^96s^26p$	$Z+79$	
27059.89	8	$4f^96s^26p$	$Z+82$	
27390.97[c]	9	–	$Z+9$	$4f^95d6s6p$
27445.90[c]	8	–	$Z+28$	$4f^95d6s6p$
27462.41[c]	7	–	$Z+12$	$4f^95d6s6p$
27474.64[c]	6	–	$Z+13$	$4f^95d6s6p$
27896.80[c]	8	–	$Z+3$	$4f^95d6s6p$
27959.98[c]	7	–	$Z+3$	$4f^95d6s6p$
27987.90[c]	6	–	$Z+16$	$4f^95d6s6p$
28158.51[c]	9	–	$Z+11$	$4f^95d6s6p$
28309.18	5	$4f^96s^26p$?	$Z+80$	$4f^96s^26p$
28326.48[c]	6	–	$Z+17$	$4f^95d6s6p$
28358.70[c]	7	–	$Z+5$	$4f^95d6s6p$
28379.82	5	$4f^96s^26p$	$Z+34$	
28433.41	10	[d]	$Z-13$	
28539.57[c]	8	–	$Z+7$	$4f^95d6s6p$
28666.31	5	–	$Z+40$	
28909.39[c]	6	–	$Z+22$	$4f^95d6s6p$
28971.42[c]	8	–	$Z+1$	$4f^95d6s6p$
28987.02	5	–	$Z+82$	$4f^96s^26p$
29291.32	8	($4f^95d6s6p$)	$Z+2$	
29465.04[c]	9	–	$Z+11$	$4f^95d6s6p$
29512.27	7	($4f^95d6s6p$)	$Z+11$	
29682.16[c]	6	–	$Z+9$	$4f^95d6s6p$
29706.72[c]	10	–	$Z+12$	$4f^95d6s6p$
29877.37	7	($4f^95d6s6p$)	$Z+10$	
30033.06[c]	5	–	$Z+6$	$4f^95d6s6p$
30106.65[c]	10	–	$Z+9$	$4f^95d6s6p$
30444.88	7	($4f^95d6s6p$)	$Z+5$	
30459.64	9	($4f^95d6s6p$)	$Z+11$	
30475.95	6	($4f^95d6s6p$)	$Z+13$	
30600.15	8	($4f^95d6s6p$)	$Z+20$	

Table 2. continued

Energy level (cm^{-1})	J	Configuration[+]	ΔT (mK)	Configuration (present work)
30621.87	7	$(4f^9 5d6s6p)$	$Z+7$	
30716.06	9	$(4f^9 5d6s6p)$	$Z+13$	
30739.79	8	$(4f^9 5d6s6p)$	$Z+4$	
31674.08	7	$(4f^9 5d6s6p)$	$Z+10$	
31820.28	8	$(4f^9 5d6s6p)$	$Z+7$	
32206.27[c]	9	–	$Z+11$	$4f^9 5d6s6p$
32392.59[c]	6	–	$Z+10$	$4f^9 5d6s6p$
33086.26	9	$(4f^9 5d6s6p)$	Z	
33753.11	8	$(4f^9 5d6s6p)$	$Z+4$	
33911.02	6	$(4f^9 5d6s6p)$	$Z+7$	
34689.19	9	$(4f^9 5d6s6p)$	$Z-3$	
34829.30	9	$(4f^9 5d6s6p)$	$Z-2$	
35354.27	5	–	$Z+80$	$4f^9 6s^2 6p$
35377.51	8	$(4f^9 5d6s6p)$	Z	
35938.74[c]	8	–	$Z+12$	$4f^9 5d6s6p$
36392.11	8	[e]	$Z-67$	
36490.07	10	[d]	$Z-11$	
36508.79[c]	7	–	$Z+17$	$4f^9 5d6s6p$
36708.15[c]	9	–	Z	$4f^9 5d6s6p$
36717.57	9	[e]	$Z-56$	
36760.64	8	$4f^{10} 6s6d$	$Z-57$	$4f^{10} 6s6d$
36807.39	8	–	$Z-20$	
36822.27	9	$(4f^9 6s^2 6p)$	$Z+80$	
36905.44	10	$4f^{10} 6s6d$	$Z-67$	$4f^{10} 6s6d$
36954.35	8	[d]	$Z-15$	
36964.54	9	$4f^{10} 6s6d$?	$Z-71$	$4f^{10} 6s6d$
37015.22	7	–	$Z-69$	$4f^{10} 6s6d$
37163.16	6	–	$Z-60$	$4f^{10} 6s6d$
37295.97	7	–	$Z-65$	$4f^{10} 6s6d$
37324.62	6	–	$Z-60$	$4f^{10} 6s6d$
37339.89	8	$4f^{10} 6s6d$?	$Z-70$	$4f^{10} 6s6d$
37591.83	9	$4f^{10} 6s6d$?	$Z-70$	$4f^{10} 6s6d$
37694.25	7	$(4f^9 5d6s6p)$	$Z+10$	
37706.12	10	$4f^{10} 6s6d$?	$Z-71$	$4f^{10} 6s6d$
37751.34	7	–	$Z-41$	
37820.22	8	$4f^{10} 6s6d$?	$Z-69$	$4f^{10} 6s6d$
37856.42	6	–	$Z-60$	$4f^{10} 6s6d$
38054.61[c]	7	–	Z	$4f^9 5d6s6p$
38264.28	7	–	$Z-15$	
38356.27[f]	8	$(4f^9 5d^2 6p)$	$Z-127$	
38674.91	9	$(4f^{10} 6s6d)$	$Z-66$	
39048.18	5	–	$Z-75$	
39120.61	9	–	$Z-74$	
39573.04[f]	10	$(4f^9 5d^2 6p)$	$Z-100$	
39750.08[f]	10	$(4f^9 5d^2 6p)$	$Z-120$	
39777.62	7	–	$Z-145$	$4f^9 5d^2 6p$

[+] Configurations in parentheses are based on our earlier studies [7, 8] and supported by Wyart's parametric calculations [14] for the $4f^9 5d6s6p$ configuration.
[a] Assumed values (see text).
[b] Values from Wyart [3].
[c] We have assigned these levels to the $4f^9 5d6s6p$ configuration and except for the levels at 35938 cm^{-1} ($J=8$), 36508 cm^{-1} ($J=7$) and 38054 cm^{-1} ($J=7$) all have been confirmed by Wyart's parametric calculations [14].
[d] May belong to the mixed $4f^9 5d6s6p$ configuration.
[e] These levels have been assigned to the $4f^{10} 6s6d$ configuration by Wyart [3]; these assignments are confirmed by us.
[f] These levels were assigned to the new configuration $4f^9 5d^2 6p$ by us [7].

$4f^96p6s^2$ configuration

Three levels at 26785 cm^{-1} ($J = 6$), 26848 cm^{-1} ($J = 7$) and 27059 cm^{-1} ($J = 8$) are assigned to the $4f^96s^26p$ configuration in [3,6] and the ΔT values for these levels are $\sim (Z + 80)$ mK (Tab. 2b). The level at 28379 cm^{-1} ($J = 5$) is assigned to the $4f^96s^26p$ configuration in [6] and Wyart [3] has assigned 91% of the f^9s^2p configuration for this level. But the observed $\Delta T = (Z + 34)$ mK does not support the high purity of 91%, it rather suggests strong configuration mixing. The level at 28309 cm^{-1} ($J = 5$) has been tentatively assigned to the $4f^96s^26p$ configuration [6]. We have evaluated $\Delta T = (Z + 80)$ mK for this level which confirms its assignment to $4f^96s^26p$. It may be however pointed out that Wyart [3] has indicated f^9s^2p configuration for this level, but has reported $\Delta T = (Z - 14)$ mK which does not agree with our ΔT value.

$4f^{10}6s6d$ configuration

There are 9 levels between 36390 and 37820 cm^{-1} (Tab. 2b) which have been assigned to the $4f^{10}6s6d$ configuration by Wyart [3]; however Martin *et al.* [6] have put 7 of these assignments as tentative and the levels at 36392 cm^{-1} ($J = 8$) and 36717 cm^{-1} ($J = 9$) are left unassigned. For a pure $f^{10}ds$ configuration the expected value of ΔT is $\sim (Z - 80)$ mK. The observed ΔT for all these 9 levels ranges from $(Z - 56)$ to $(Z - 71)$ mK (see Tab. 2b) which justifies their assignments by Wyart [3].

$4f^95d^26p$ configuration

Our studies [7] were the first to identify the configuration $4f^95d^26p$ in Dy I and three levels at 38356 cm^{-1} ($J = 8$), 39573 cm^{-1} ($J = 10$) and 39750 cm^{-1} ($J = 10$) were assigned by us to the new configuration $4f^95d^26p$ on the basis of the observed ΔT values. The present study confirms the earlier ΔT values of these levels and our assignments.

3.1.3 Unassigned even levels and suggested possible configurations

f^9dsp configuration

There are a large number of even levels of Dy I above 27390 cm^{-1} (Tab. 2b) which do not have any configuration assignment in [6]. On the basis of our earlier studies [7,8], we expect that the high even levels combining with low odd levels of $4f^95d6s^2$ and showing ΔT values between $(Z - 10)$ and $(Z + 10)$ mK should belong to the $4f^95d6s6p$ configuration. There are 24 levels which have been presently assigned to the $4f^95d6s6p$ configuration and these levels are marked with the superscript “c” in Table 2b. Our assignments of these levels to $4f^95d6s6p$ are also supported by the theoretical calculations of Wyart [14].

f^9s^2p configuration

There is one unassigned level at 28987 cm^{-1} ($J = 5$) for which we have evaluated $\Delta T = (Z + 82)$ mK which shows that this level could belong to the f^9s^2p configuration. Wyart [14] has calculated a level at 28997 cm^{-1} ($J = 5$) for the $4f^96s^26p$ configuration with calculated value for $g = 1.076$, but this does not agree with the experimental value ($g = 1.317$) for the known level at 28987 cm^{-1}.

The level at 28666 cm^{-1} ($J = 5$) has $\Delta T = (Z + 40)$ mK and this level should belong predominantly to the f^9s^2p configuration.

$f^{10}ds$ configuration

Five unassigned levels at 37015 cm^{-1} ($J = 7$), 37163 cm^{-1} ($J = 6$), 37295 cm^{-1} ($J = 7$), 37324 cm^{-1} ($J = 6$) and 37856 cm^{-1} ($J = 6$) have ΔT values ranging from $(Z - 60)$ mK to $(Z - 70)$ mK; we suggest $f^{10}sd$ as the possible configuration to all these levels. The level at 38674 cm^{-1} ($J = 9$) was assigned to the $f^{10}sd$ configuration in our earlier work [7] which is confirmed in the present study.

f^9d^2p configuration

The level at 39777 cm^{-1} ($J = 7$) has $\Delta T = (Z - 145)$ mK and we assign this level to f^9d^2p configuration.

3.1.4 ΔT values of odd energy levels of Dy I

Out of the 68 odd levels listed in Table 3, ΔT values for 24 odd levels are being reported for the first time. As could be seen from column 3 of the table, many high-lying odd levels lying above 35000 cm^{-1} have only tentative configuration assignments. The ΔT values of low even levels (Tab. 2a) have been used to evaluate the ΔT values of high odd levels (Tab. 3).

3.1.5 Comments on the configuration assignment to odd energy levels

The average ΔT values of different odd configurations of Dy I evaluated in the present study and the earlier studies [3,7,8,15–17] can be summarized as

$$\Delta T(4f^95d6s^2) \sim (Z + 90)\ \text{mK},$$
$$\Delta T(4f^95d^26s) \sim (Z + 10)\ \text{mK},$$
$$\Delta T(4f^{10}6s6p) \sim (Z - 70)\ \text{mK},$$
$$\Delta T(4f^{10}5d6p) \sim (Z - 155)\ \text{mK}.$$

$4f^95d6s^2$ configuration

There are 22 levels assigned to the $4f^95d6s^2$ configuration [6] encountered in the present work (Tab. 3). The observed ΔT values of these levels, except for one level at 18339 cm^{-1} ($J = 7$), are in the range from $(Z + 76)$ to $(Z + 90)$ mK. The level at 18433 cm^{-1} ($J = 7$) has been assigned to the $4f^{10}6s6p$ configuration [3,6] and its observed ΔT value is $(Z + 19)$ mK, whereas the expected ΔT value

Table 3. ΔT (^{160}Dy-^{164}Dy) for the odd-parity energy levels of Dy I encountered in the present study. Energy levels and configurations are from [6].

Energy level (cm^{-1})	J	Configuration	ΔT (mK)	Configuration (present work)
7565.60	8	$4f^95d6s^2$	$Z+79$	
8519.20	7	$4f^95d6s^2$	$Z+82$	
9990.95	9	$4f^95d6s^2$	$Z+86$	
10088.80	6	$4f^95d6s^2$	$Z+80$	
11673.49	6	$4f^95d6s^2$	$Z+80$	
12007.10	8	$4f^95d6s^2$	$Z+81$	
12298.56	5	$4f^95d6s^2$	$Z+75$	
12655.13	7	$4f^95d6s^2$	$Z+83$	
13495.92	9	$4f^95d6s^2$	$Z+90$	
14153.49	5	$4f^95d6s^2$	$Z+80$	
14625.64	8	$4f^95d6s^2$	$Z+81$	
15194.83	7	$4f^95d6s^2$	$Z+79$	
15567.38	8	$4f^{10}6s6p$	$Z-65$	
15972.35	9	$4f^{10}6s6p$	$Z-67$	
16693.87	7	$4f^{10}6s6p$	$Z-60$	
16733.20	8	$4f^{10}6s6p$	$Z-69$	
17513.33	10	$4f^{10}6s6p$	$Z-66$	
17687.90	7	$4f^95d6s^2$	$Z+79$	
17727.15	9	$4f^{10}6s6p$	$Z-70$	
18021.89	8	$4f^{10}6s6p$	$Z-70$	
18339.80	7	$4f^95d6s^2$	$Z+10$	
18433.76	7	$4f^{10}6s6p$	$Z+19$	
18472.71	8	$4f^95d^26s$	Z	
18528.55	7	$4f^95d^26s$	Z	
18711.93	6	$4f^{10}6s6p$	$Z-60$	
18857.04	7	$4f^95d6s^2$	$Z+76$	
19092.30	8	$4f^95d6s^2$	$Z+81$	
19304.26	6	$4f^95d^26s$	$Z-5$	
19557.83	9	$4f^95d^26s$	$Z-3$	
19688.59	8	$4f^95d6s^2$	$Z+85$	
19797.96	10	$4f^95d^26s$	$Z-1$	
20341.32	8	$4f^{10}6s6p$	$Z-66$	
20485.40	7	$4f^95d6s^2$	$Z+84$	
20766.29	7	$4f^{10}6s6p$	$Z-60$	
21899.22	8	$4f^{10}6s6p$	$Z-68$	
22633.23	6	$4f^95d6s^2$	$Z+82$	
23464.02[a]	6	$4f^95d6s^2$	$Z+65$	
23687.87	6	$4f^{10}6s6p$	$Z-51$	
23877.75	8	$4f^{10}6s6p$	$Z-32$	
24040.59	6	$4f^95d6s^2$	$Z+82$	
24906.86	7	$4f^{10}6s6p$	$Z-70$	
24999.58	8	$4f^{10}6s6p$	$Z-60$	
25012.21	8	$4f^95d^26s$	$Z+2$	
26135.21	5	$4f^95d6s^2$	$Z+81$	
27190.74	6	$4f^95d^26s$	$Z-10$	
27199.20	6	$4f^{10}6s6p$	$Z-53$	
27427.08	7	$4f^{10}6s6p$	$Z-64$	
28923.05	4	$4f^{10}6s6p$	Z	
29447.11	6	$4f^{10}6s6p$	Z	
35221.27[b]	9	$4f^{10}5d6p$?	$Z-105$	$4f^{10}5d6p$
35945.02[b]	10	$4f^{10}5d6p$?	$Z-134$	$4f^{10}5d6p$
36316.42[b]	7	$4f^{10}5d6p$?	$Z-85$	$4f^{10}5d6p$
36640.90[b]	10	$4f^{10}5d6p$?	$Z-49$	$4f^{10}5d6p$

Table 3. continued

Energy level (cm^{-1})	J	Configuration	ΔT (mK)	Configuration (Present work)
37090.46[b]	9	$4f^{10}5d6p$?	$Z-86$	$4f^{10}5d6p$
37721.06[b]	8	$4f^{10}5d6p$?	$Z-65$	$4f^{10}5d6p$
37836.50[b]	9	$4f^{10}5d6p$?	$Z-70$	$4f^{10}5d6p$
38019.12	10	–	$Z-70$	
38202.48[b]	8	$4f^{10}5d6p$?	$Z-85$	$4f^{10}5d6p$
38247.38[b]	10	$4f^{10}5d6p$?	$Z-124$	$4f^{10}5d6p$
38297.52[b]	10	$4f^{10}5d6p$?	$Z-80$	$4f^{10}5d6p$
38342.48	7	$4f^{10}5d6p$?	$Z-32$	
38563.33[b]	9	$4f^{10}5d6p$?	$Z-95$	$4f^{10}5d6p$
39135.32[b]	8	$4f^{10}5d6p$?	$Z-136$	$4f^{10}5d6p$
39182.40[c]	10	$4f^{10}5d6p$?	$Z-110$	
39602.47[c]	9	$4f^{10}5d6p$?	$Z-85$	
40030.48[b]	9	$4f^{10}5d6p$?	$Z-70$	$4f^{10}5d6p$
40605.98	8	–	$Z-36$	
41656.46	6	$4f^{10}5d6p$?	$Z+17$	

[a] Level not listed in Martin *et al.* [6], taken from Wyart [3].

[b] The tentative assignments of these levels are confirmed in the present study.

[c] Assignment confirmed in our earlier study [7].

for a level of pure $f^{10}sp$ configuration is $\sim(Z-70)$ mK. This level at 18433 cm^{-1} ($J=7$) has been assigned to $4f^95d6s^2$ by Griffin *et al.* [17]. There is one level at 18339 cm^{-1} ($J=7$) belonging to the f^9ds^2 configuration with same value of $J=7$ having $\Delta T=(Z+10)$ mK, whereas the expected ΔT value is $\sim(Z+80)$ mK for a level of pure f^9ds^2 configuration. The isotope shifts in these two levels which have strong configuration interaction have been already discussed in [17].

On the basis of parametric calculations, Wyart [3] has suggested 61% f^9ds^2 and 36% f^9d^2s composition for the level at 18857 cm^{-1} ($J=7$). For this level the observed ΔT value is $(Z+76)$ mK; a value one expects for an almost pure level of f^9ds^2 configuration. Similarly the level at 18528 cm^{-1} ($J=7$) has $\Delta T=Z$ mK and the composition proposed for this level is 33% f^9ds^2 and 64% f^9d^2s [3] which is not reflected in the observed ΔT value. For the level at 23464 cm^{-1} ($J=6$) Wyart [3] has given a composition of 64% f^9ds^2 and 35% f^9d^2s and our observed value of $\Delta T=(Z+65)$ mK points to the suggested configuration mixing [3]. It may be, however, pointed out that this level of Dy I does not appear in the energy level listing of Martin *et al.* [6].

$4f^{10}6s6p$ configuration

We have encountered 20 levels in the present study (Tab. 3) which are assigned to the $4f^{10}6s6p$ configuration [3,6]; out of these, 14 levels have ΔT values ranging from $(Z-60)$ to $(Z-72)$ mK confirming their configuration assignments. The level at 23687 cm^{-1} ($J=6$) has been assigned to the 38% f^9ds^2 and 61% $f^{10}sp$ mixed configuration [3] and the observed $\Delta T=(Z-51)$ mK confirms this. The level at 23877 cm^{-1} ($J=8$) has been assigned to the almost pure (98%) $f^{10}sp$ configuration [3], but the observed value of ΔT is $(Z-32)$ mK. This level belongs to the ^1P multiplet and it has been observed earlier by Griffin *et al.* [17] and also by us that the ^1P levels of the "sp" configuration show abnormally small isotope shifts. The level at 18433 cm^{-1} ($J=7$) has been assigned to 89% $f^{10}sp$ and 11% f^9ds^2 [3] but the observed $\Delta T=(Z-19)$ mK does not confirm this suggested composition (we have already discussed this level in the previous section on the $4f^95d6s^2$ configuration). The level at 28923 cm^{-1} ($J=4$) has been assigned 99% pure $f^{10}sp$ configuration [3], but its observed ΔT value of Z mK does not support this. The level at 29446 cm^{-1} ($J=6$) has been assigned to 100% $f^{10}sp$ [3], but instead of the expected ΔT value of $(Z-70)$ mK we have observed $\Delta T=Z$ mK for this level.

$4f^95d^26s$ configuration

In the present study only seven levels of the $4f^95d^26s$ configuration are involved (Tab. 3). All the seven levels have ΔT values between $(Z-10)$ and $(Z+2)$ mK, confirming their assignments.

$4f^{10}5d6p$ configuration

There are 16 levels lying between 35221 cm^{-1} and 40605 cm^{-1} having ΔT values mostly between $(Z-70)$ and $(Z-136)$. Wyart [3] has left these levels unassigned, but Martin *et al.* [6] have tentatively assigned these levels to the $4f^{10}5d6p$ configuration and our ΔT values in general confirm these assignments. The levels at 38342 cm^{-1} ($J=7$) and 36640 cm^{-1} ($J=10$) have been tentatively assigned to $f^{10}dp$ [6], but their ΔT values of $(Z-32)$ mK and $(Z-49)$ mK, respectively, do not support this tentative assignment. For the level at 41656 cm^{-1} $(J=6)$ we have obtained $\Delta T=(Z+17)$ mK which does not support its tentative assignment to the $4f^{10}5d6p$ configuration [6], however the ΔT value is based on the isotope shift measurement in only one line. The unassigned level at 38019 cm^{-1} $(J=10)$ with $\Delta T=(Z-70)$ mK may belong to a mixed $f^{10}dp$ configuration.

4 Conclusion

The present study has provided isotope shift data in 221 spectral lines of Dy I; 164 lines have energy level classifications and 57 lines, although assigned to Dy I, are not yet classified. Isotope shifts in 173 lines are being reported for the first time. The present data has enabled us to evaluate term isotope shifts (ΔT) of odd- and even-parity energy levels of Dy I; ΔT values for 36 even and 24 odd levels are being reported for the first time. Most of the high-lying levels of Dy I have either tentative assignments or are unassigned. The present study has enabled us to confirm most of these tentative assignments. Many high odd levels above 35800 cm^{-1} which were tentatively assigned to the $4f^{10}5d6p$ configuration have been mostly confirmed in the present study. The present study also enabled us to check some of the levels for their composition of configuration mixing reported on the basis of parametric calculations. ΔT values obtained for the unassigned energy levels of Dy I in this study have been interpreted. 24 unassigned even levels have been assigned to the $4f^{9}5d6s6p$ configuration.

References

1. J.G. Conway, E.F. Worden, J. Opt. Soc. Am. **61**, 704 (1971).
2. P. Camus, K. Masmoudi, Spectrochim. Acta B **28**, 79 (1973).
3. J.F. Wyart, Thesis, University of Paris (1973).
4. J.F. Wyart, C. R. Acad. Sci. B **273**, 763 (1971).
5. J.F. Wyart, Physica **75**, 371 (1974).
6. W.C. Martin, R. Zalubas, L. Hagan, *Atomic Energy Levels: The Rare Earth Elements*, NSRDS-NBS 60 (National Bureau of Standards, Washington, 1978).
7. S.A. Ahmad, A. Venugopalan, G.D. Saksena, Spectrochim. Acta B **37**, 181 (1982).
8. S.A. Ahmad, A. Venugopalan, G.D. Saksena, Spectrochim. Acta B **38**, 639 (1983).
9. R. Neugart, K. Wendt, S.A. Ahmad, W. Klempt, C. Ekstrom, Hyperfine Interact. **15/16**, 181 (1983).
10. W.J. Childs, V. Pfeufer, L.S. Goodman, J. Opt. Soc. Am. B **1**, 22 (1984).
11. V. Pfeufer, W.J. Childs, L.S. Goodman, J. Opt. Soc. Am. B **1**, 34 (1984).
12. M. Wakasugi, T. Horiguchi, W.G. Jin, H. Sakata, Y. Yoshizawa, J. Phys. Soc. Jpn. **59**, 2700 (1990).
13. R.J. Lipert, S.L. Lee, J. Phys. B **57**, 373 (1993).
14. J.F. Wyart, Private Communication (1983).
15. A.R. Striganov, A.F. Golovin, P. Gerasimova, Opt. Spectrosc. **14**, 3 (1963).
16. J.S. Ross, J. Opt. Soc. Am. **62**, 548 (1972).
17. D.C. Griffin, J.S. Ross, J. Opt. Soc. Am. **62**, 571 (1972).
18. J.W.M. Dekker, P.F.A. Klinkenberg, J.F. Langkemper, Physica **39**, 393 (1968).